SOME PROPERTIES OF QUASI-DEGREES

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Tennenbaum (see [3, p.159]) defined the notion of Q-reducibility on sets of natural numbers as follows: a set A is *quasi-reducible* (*Q-redusible*) to a set B (in symbols: $A \leq_Q B$) if there exists a computable function f such that for every $x \epsilon \omega$ (where ω denotes the set of natural numbers),

$$x \epsilon A \iff W_{f(x)} \subseteq B.$$

Recently Batyrshin [1] proved that there exists a noncomputable c.e. Q-degree which contains only one c.e. m-degree. We improve this result by showing that every noncomputable c.e. Q-degree contains a c.e. perfect set (see [2]).

Our notation and terminology are standard and can be found in [3].

Theorem 1. Every noncomputable c.e. Q-degree contains a c.e. perfect set.

Corollary 1. A noncomputable c.e. Q-degree consists a single c.e. m-degree if and only if it consists of a single c.e. 1-degree.

Corollary 2. Every noncomputable c.e. Q-degree contains a c.e. m-degree consisting of only one 1-degree.

Corollary 3. There is a noncomputable c.e. Q-degree containing a single c.e. 1-degree.

Theorem 2. Let K be a creative set. Then there is a $\Sigma_2^0 - \Delta_2^0$ set B which is Q-incomparable with K and for all c.e. sets W, if $W \leq_Q B$ then $W \leq_Q \emptyset$.

Corollary 1. For every Π_2^0 set A there is a $\Sigma_2^0 - \Delta_2^0$ set B such that A and B are Q-incomparable and for all c.e. sets W, if $W \leq_Q A$ and $W \leq_Q B$ then $W \leq_Q \emptyset$.

References

[1] I.I.Batyrshin, Irreducible, singular, and contiguous degrees, Algebra and Logic, 56 (2017), 181-196.

[2] Yu.L.Ershov, Positive equivalences, Algebra and Logic, 10 (1971), 378-394.

[3] H.Rogers, Theory of recursive functions and effective computability. McGraw-Hill Book Co., New York, 1967.