Integral representation of Wiener functional of a difference of arguments

O. Purtukhia
E-mail: omar.purtukhia@tsu.ge; o.purtukhia@gmail.com
Ivane Javakhishvili Tbilisi State University, A.Razmadze Mathematical Institute;
Faculty of Exact and Natural Sciences, Department of
Mathematics, 3 Chavtchavadze Ave, Tbilisi 0179, Georgia

As is well-known the Clark formula yields a fairly abstract result for stochastic integral representtation of Wiener functional. As for the explicit expression of the integrand, it can only be obtained in special cases which we discuss below. Most of the general research on constructive integral representtation has been within Malliavin calculus. Here the constructive integral representation is based on the Malliavin (stochastic) derivative and in Wiener case is known as the Clark-Ocone formula (see, Ocone, 1984) and in the case of normal martingales for functionals from the class $D_{2,1}^{M}$ -- as the Clark-Haussmann-Ocone formula (see, Ma, Protter and Martin, 1998). We (Purtukhia, 2003) have introduced the space $D_{p,1}^{M}$, $1<p<2$ and extended the Clark-Haussmann-Ocone formula for functionals from this space. But in all the above-mentioned cases, on the one hand, stochastic smoothness is required, and on the other hand, even in the case of smoothness, the calculations of the Malliavin derivative and the conditional mathematical expectation are rather complicated.

Absolutely different method for finding of integrand, for the so called maximum functionals, was offered by Shyriaev, Yor and Graversen (2003, 2006), which was based on using of Ito's (generalized) formula and Levy's theorem for associated to functional Levy's martingale. We (Purtukhia, Jaoshvili, 2009) introduced the new construction of stochastic derivative of Poisson functional and established the explicit expression for the integrand of Clark representation.

The class of functional to which the Clark-Haussmann-Ocone formula can be applied is, however, limited by the condition that the functional must be Malliavin differentiable. We (Glonti and Purtukhia, 2014) considered case when functional is not Malliavin differentiable, but from its conditional mathematical expectation one can to select a Malliavin differentiable subsequence (with respect to $t\in[0,T)$) and generalized the Clark-Haussmann-Ocone formula. Here we consider the functional of a difference of arguments $h(W_T-W_S)I_{\{W_T-W_S<K\}}$, which is not Malliavin differentiable and establish for it the stochastic integral representation formula with explicit form of integrand.

Theorem. If $h\in C^1$ and there exists $\lambda>0$ such that $\lim_{x\to\infty} h(x)\exp\{-\lambda x^2\}=0$, then the stochastic integral representation is fulfilled:

$$h(W_T-W_S)I_{\{W_T-W_S<K\}}=E[h(W_T-W_S)I_{\{W_T-W_S<K\}}]+$$

$$+\frac{1}{\sqrt{2\pi}}\int_0^T\sqrt{\frac{T-S}{T^2-(T-S)t}}\int_{-\infty}^{K}h'(x)\exp\left\{-\frac{[Tx-(T-S)W_t]^2}{2[T^2(T-S)-(T-S)^2t]}\right\}dxdW_t .$$

Corollary.

$$h(W_T)I_{\{W_T<K\}}=E[h(W_T)I_{\{W_T<K\}}]+\int_0^T\frac{1}{\sqrt{2\pi(T-t)}}\int_{-\infty}^{K}h'(x)\exp\left\{-\frac{(x-W_t)^2}{2(T-t)}\right\}dxdW_t .$$