On the convergence rate of Cesaro means of negative order of series with respect to block-orthonormal systems

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Let $\{N_k\}$ be increasing sequences of natural numbers and

$$\Delta_k = \left(N_k,\ N_{k+1}\right], \quad (k \ge 1) \ .$$

Let $\{\varphi_n\}$ be a system of functions from $L^2(0,1)$. The system $\{\varphi_n\}$ will be called a Δ_k-orthonormal system if $\|\varphi_n\|_2 = 1, \quad n = 1,2,...$ and $\left(\varphi_i, \varphi_j\right) = 0$, for $(i,j) \in \Delta_k$, $i \ne j$, $(k \ge 1)$.

Below we shall consider the (c,α), $(-1 < \alpha < 0)$ summability a.e. of series with respect to Δ_k-orthonormal systems. In particular, it was established that if $\{\varphi_n\}$ be a system of normed functions from $L^2(0,1)$, then condition $\sum_{n=1}^{\infty} a_n^2 n^{-2\alpha} < \infty$ guarantees the summability a. e. on (0,1) by the $(c,\alpha), (-1 < \alpha < -\frac{1}{2})$ methods of the series $\sum_{n=1}^{\infty} a_n \varphi_n(x)$. If $-\frac{1}{2} \le \alpha < 0$ and $\{N_k\}$ satisfy the conditions: $\frac{N_k}{k}$ is nondecreasing and $k = O(N_k - N_{k-1})^{-\alpha}$, $(k \to \infty)$, then for arbitrary $\Delta_k = (N_k, N_{k+1}]$-ONS $\{\varphi_n\}$ condition $\sum_{n=1}^{\infty} a_n^2 n^{-2\alpha} < \infty$ guarantees the summability a.e. on (0,1) by the (c,α) of the series $\sum_{n=1}^{\infty} a_n \varphi_n(x)$.

Let $\sigma_n^{\alpha}(x)$ be the (c,α), $(-1 < \alpha < 0)$ means of the series $\sum_{n=1}^{\infty} a_n \varphi_n(x)$ with respect to Δ_k-orthonormal systems. We investigated the convergence rate of $\Delta_n^{\alpha}(x) = \sigma_n^{\alpha}(x) - \lim_{n\to\infty} \sigma_n^{\alpha}(x)$ under the condition $\sum_{n=1}^{\infty} a_n^2 n^{-2\alpha} \omega(n) < \infty$ and established the conditions on $\omega(n)$, when we have: $\Delta_n^{\alpha}(x) = o\left(\frac{1}{\sqrt{\omega(n)}}\right), (n \to \infty)$ a. e. and this estimate is exact.