For a fixed $T > 0$ we consider the stochastic process $(X_t)_{t\geq 0}$ that satisfies

$$X_t = X_0 + \int_0^t b(X_s)ds + f(t) + W_t, \quad t \in [0,T],$$

Where $(W_t)_{t\in[0,T]}$ is a standard one-dimensional Brownian motion defined on some $(\Omega, \mathfrak{F}_t, \mathbb{p})$ filtered probability space, $f \in C^1[0,T]$ such that $f(0) = 0$, and $b: \mathbb{R} \to \mathbb{R}$ is Lipschitz continuous such that

$$|b(x)| \leq M(|x| + 1), \qquad |b(x) + b(x)| \leq K|x - y|, \quad \forall x, y \in R,$$

For some constants M, K. Here $C^1([0,T])$ denotes the space of real-valued continuously differentiable functions equipped with the usual norm $\|.\|_{C^1([0,T])}$. ν is a distribution of X_0.

Consider 1 as a threshold, we moreover suppose that $X_0 < 1$ and define the stopping time

$$\tau = \inf\{t > 0: X_{t\wedge T} \geq 1\}$$

To be the first time that X_t reaches the level 1. For a given ν we are interested in two related densities

$$\mathfrak{p}^\nu(t) = \frac{d}{dt}\mathbb{p}^\nu(\tau \leq t), \qquad t \in [0,T],$$

i.e. the density of τ, and

$$p^\nu(t,y) = \frac{d}{dt}\mathbb{p}^\nu(X_t \leq y, t < \tau), \quad t \in [0,T], \quad y \in (-\infty, 1]$$