On some numerical experiments on symmetric games

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In its 80 years of existence, the field of linear programming becamed the most powerful tool in theoretical computer science for algorithm design. Nowadays, LP effectively solves integer programming, graph theory and other problems and is a widely used technique for designing approximation algorithms for complex tasks. Besides, for many computational problems, proofs of efficient algorithm existence are LP based.

Linear programming has several equivalent formulations. We are experimenting with symmetric games. In this case it is possible using penalty functions together with general-purpose unconstrained minimimization solvers like l-bfgs (see [1]) of modified heavy ball (see [2]), yet get algorithm, competetive to open source or comercial LP-solvers. In this approach, the main problem lies in constraints of the sign of variables and we have two-stage solution to this problem. Consider this issue roughly but in some details.

Solving symmetric game with skew symmetric matrix A is equivalent to solution of the following system

$$\begin{cases} A_0 x \le 0, \ldots, A_{n-1} x \le 0 \\ x_0 + \cdots + x_{n-1} = 1 \\ x_0 \ge 0, \ldots, x_{n-1} \ge 0 \end{cases}.$$

Let us take usual default feasibility tolerance 1E-6.

On the first stage, we solve it with penalty:

$$P_1(x) = \frac{1}{3}\left(\sum_{i=0}^{n-1} (A_i x \le 0)?0:(A_i x)^3 + 100 \cdot \sum_{i=0}^{n-1} (x_i \ge 0)?0:(-x_i)^3 + (x_0 + \cdots + x_{n-1} - 1)^3 \right)$$

But with tolerance 1E-7. Then we renew initial iterate by substitution of the variables:

$$\texttt{if}(\, x_i > 0 \,) \quad x_i = \log(x_i)/10\,;$$
$$\texttt{else} \quad x_i = \log(1\mathrm{E}-10);$$

and resolve it with default tolerance using

$$P_2(t) = \frac{1}{3}\left(\sum_{i=0}^{n-1} A_i x \le 0?0:(A_i x)^3 + \left(e^{10t_0} + \cdots + e^{10t_{n-1}} - 1\right)^3 \right).$$

Now, second stage has little overhead and after restoring initial variables our solution has non-negative variables, only.

References

1. Jorge Nocedal Stephen J. Wright. Numerical Optimization, Second Edition, Springer, 2006.
2. Koba Gelashvili, Irina Khutsishvili, Luka Gorgadze, Lela Alkhazishvili. Speeding up the convergence of the Polyak's Heavy Ball algorithm (in print)