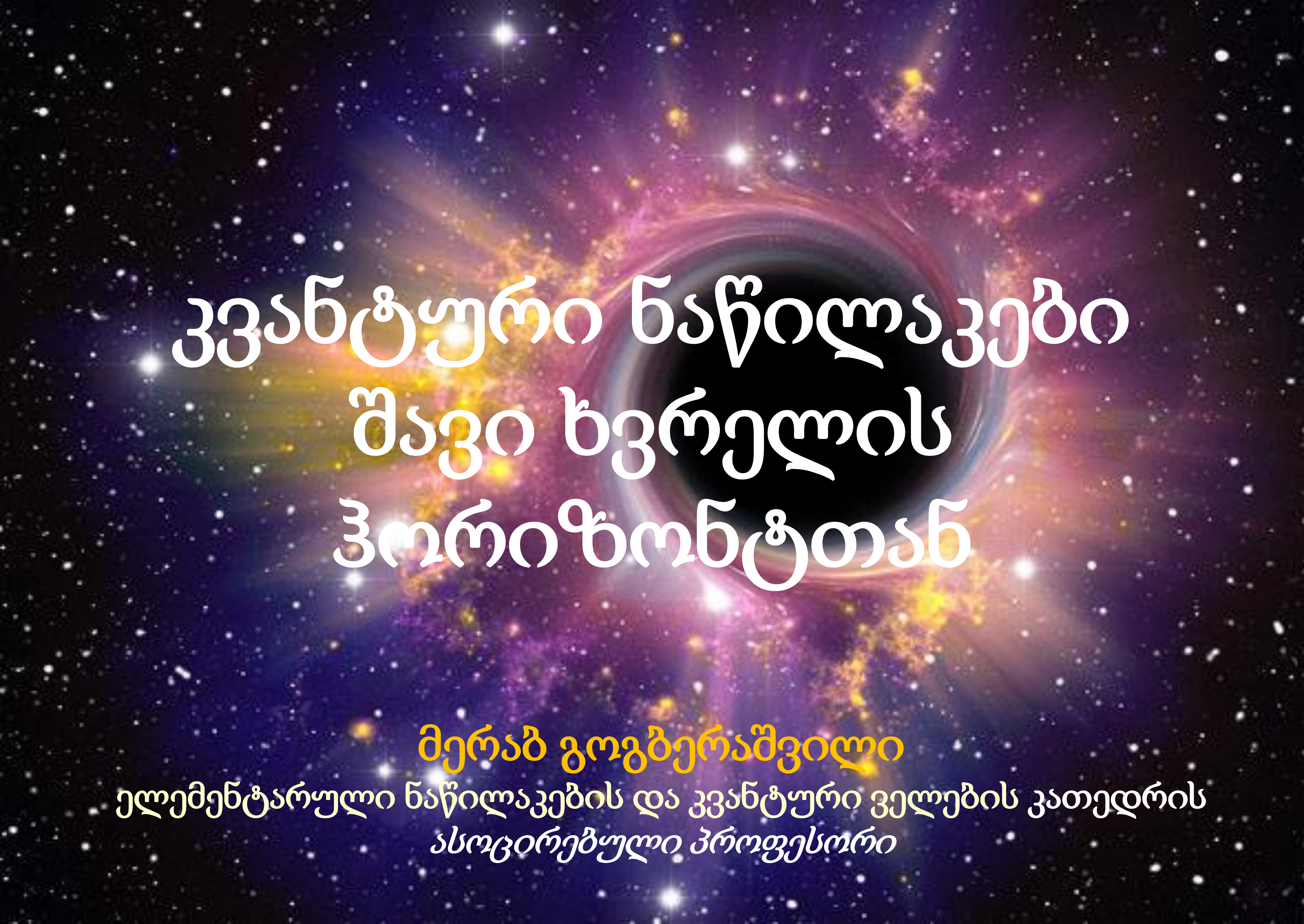

კვანტური ნაწილაკები შავი ხვრელის ჰორიზონტთან

მერაბ გოგბერაშვილი

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მოხსენების გეგმა:

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Singularity Problem

Spacetime singularities are an inevitable feature for most of the solutions of the **Einstein** equations. According to the weak censorship conjecture, the naked singularities are always be screened by an observer horizon.

Geometry does not exist separately from matter and the most accepted criterion of existing of singularities is the geodesic incompleteness. In this approach it is assumed that so called coordinate singularities can be avoided "repairing" geodesics by singular coordinate transformations.

The most important singular solution of the **Einstein** equations is the **Schwarzschild** metric and **BH**s. The assumption that quantum particles can freely fell through the **BH** event horizon contradicts a unitary quantum theory and leads to the another problem - the **BH** information paradox.

Close to singularities one should use a theory of quantum gravity, where naked singularities cannot exist. Close to a horizon gravitational field becomes strong and it is insufficient to explore this region by classical geodesic (**Hamilton-Jacobi**) equations. Instead one should use at least quasi-classical approximation and obtain classical trajectories from the particles wavefunctions in geometrical-optical limit (eikonal approximation) .

Eikonal Approximation

Einstein's gravity doesn't care about spin and close to a horizon particles can be described by the **Klein-Gordon** equation in a curve space-time,

$$\left[\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\right)+m^2\right]\Phi(x^\alpha)=0\,.$$

The wavefunction associated with the classical motion can be written as:

$$\Phi=Ae^{iS}\,,$$

the relativistic **Hamilton**'s principal function usually is used in the definition of 4-momentum, $p^\alpha\sim\partial^\alpha S$. Then **Klein-Gordon** equation gives the system:

$$A\Box S+2\,\partial_\nu S\partial^\nu A=0\,,$$

$$\Box A-A\,\partial_\nu S\partial^\nu S+m^2A=0\,.$$

To obtain the geodesic (**Hamilton-Jacobi**) equation one should consider the week gravitational field and short wavelength limit ($\Box S\to 0$), and neglect variations of the wave amplitude ($\partial^\nu A\to 0$). From the first condition it follows that the eikonal phase (**Hamilton**'s principal function) has the form:

$$S\sim p_\nu x^\nu$$

and p^α obeys the **Hamilton-Jacobi** equation,

$$g_{\alpha\beta}p^\alpha p^\beta-m^2=0\,.$$

Waves on Schwarzschild

We consider a non-rotating, uncharged **BH** with the line element,

$$ds^2 = f(r)\,dt^2 - \frac{dr^2}{f(r)} - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\,,$$

where

$$f(r) = 1 - \frac{2M}{r}. \qquad (c = \hbar = G = 1)$$

We study scalar waves,

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\,g^{\mu\nu}\partial_\nu)\Phi(x^\alpha) = 0,$$

since in the eikonal approximation the polarization tensors are parallel-transported along the null geodesic and can be regarded to be constant.

The **Schwarzschild** space-time is highly symmetric, so we can separate the variables

$$\Phi(x^\alpha) \sim \psi(t,r)Y_{lm}(\theta,\varphi)\,,$$

where Y_{lm} are spherical harmonics. For the waves with zero angular momentum $l = 0$ we have

$$\left[r^2\partial_t^2 - f\,\partial_r\left(r^2 f\partial_r\right)\right]\psi(t,r) = 0\,.$$

Boundary Conditions

After separation of the variables

$$\psi(t,r) = \frac{1}{r} T(t) R(r) \ ,$$

the **Klein-Gordon** equation on **Schwarzschild** gives the system of equations:

$$\partial_t^2 T = C T \ ,$$

$$f^2 \partial_r^2 R + \frac{2Mf}{r^2} \partial_r R - \left(C + \frac{2Mr}{r^3} \right) R = 0 \ .$$

The boundary conditions for the system at $f(r) \to 0$ is usually settled assuming the existence of a horizon crossing, ingoing radial waves,

$$T(t)|_{f\to 0} \sim e^{\pm i\omega t}, \qquad R(r)|_{f\to 0} \to \sim e^{\pm i\omega r^*},$$

where the so called **Regge-Wheeler** tortoise coordinate is introduced,

$$r^* = \int \frac{dr}{f} = r + 2M \, ln \left(\frac{r}{2M} - 1 \right)$$

This assumption corresponds to the negative separation constant,

$$C = -\omega^2 < 0 \ .$$

However, the used coordinates are singular and the solution does not obey the initial equation, due to the appearance of the **Dirac** delta function in the second derivatives of the tortoise coordinate function r^* at $r = 2M$.

Approximate Solution

At the **BH** horizon let us use $f(r)$ as an independent variable (instead of r),

$$f^2(1-f)^4\,R'' + f(1-f)^3(1-3f)\,R' - \left[4M^2C + f(1-f)^3\right] R = 0\ .$$

Close to the horizon the radial wave function should satisfy the condition:

$$R(r)|_{f\to 0} \to 0\ .$$

We will look for the solution to this equation in the form:

$$R(f) = \sum_{i=1}^{\infty} a_i f^i$$

The first equation of the system has the form:

$$f(1-f)^3(1-3f)a_1 - \left[4M^2C + f(1-f)^3\right]a_1 f \approx (1-4M^2C)a_1 f = 0\ .$$

So the separation constant appears to be positive

$$C = 1/4M^2 > 0\ ,$$

and close to the **BH** horizon we have the real-valued exponential solution:

$$T(t) = T_0 e^{\pm t/2M}\ .$$

This means that in the **Schwarzschild** coordinates of a distant observer the wave function has the form:

$$\psi(t,r) = \frac{e^{\pm t/2M}}{r}\sum_{i=1}^{\infty} a_i\left(1-\frac{2M}{r}\right)^i\ .$$

Relativistic Lensing

Emission mechanisms of observed **GW**s, **GRB**s and **FRB**s are still doubtful. We want to explains these unusually strong signals by means of the amplification of the gravitationally strongly lensed waves by **BH**s.

Weak gravitational lensing is well studied; however, wave deflection remains a difficult subject in the strong gravitational field. The most promising focusing agents for any kind of waves, **BH**s, often are located at the centers of galaxies and are surrounded by dust clouds. So there are observation problems of the **BH** lensing, because of spurious radiation and large extinctions of waves by accreting materials.

In the case of electromagnetic radiation it is expected that, due to the absorptions (larger for smaller wavelength), only the short duration relativistic lensing signals in the forms of radio (largest wavelength) and gamma (most energetic) waves could escape surrounding **BH** dust. For **GW**s, dust clouds and noise do not present an obstacle. These increases chances to observe several relativistic images of the same **GW** source, since waves passing close to the **BH** can loop around it before reaching an observer. Then, unlike the case with the single **GRB**s and **FRB**s impulses, a distant observer will detect periodic **GW** signals imitating **LIGO** events.

BHs Atmosphere

We use analogies from the models of classical interpretation of **Schrödinger**'s wave function, where variations of the amplitude $\mathbf{A}$ is taking into account by introduction of so called quantum potential. In the thermodynamic approach, the variation of the distribution density of a moving particle is modeled by a resistance of the ensemble in the form of the heat flow. Appearance of the heat flow close to the **BH**s horizon is analogous to the '**firewall**' conjecture, the existence of energetic curtain at the event horizon, as if **BH**s have effective '**quantum atmospheres**' in the range of $2M \leq r \leq 3M$. In the quasi-classical approximation one can still describe trajectories of particles close to a **BH** by ordinary geodesic equations, but with extra potential.

Relativistic invariant vacuum energy in **GR** is known under the name of cosmological constant. Solution of the spherically symmetric Einstein equations with the cosmological term provided that $f(r)$ is replaced by:

$$F(r) = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2 ,$$

The constant Λ should be fixed in the way which enables us to use **Klein-Gordon** equation close to the **Schwarzschild** horizon and at the same time to cancel apparent energy non-conservation for a distant observer.

Effective AdS Space

The modified **Klein-Gordon** equation as the series of $F(r)$ takes the form:

$$F(1-F)^3(1-3F)a_1 - \left[4M^2C + F(1-F)^3\right]a_1 F =$$
$$= \left[(1-F)^3(1-4F) - 4M^2C\right]a_1 F = 0\,.$$

We require validity of this equation at the horizons of $F(r)$ and $f(r)$ simultaneously, i.e. at

$$F = 0\,, \qquad f = 0 \leftrightarrow F = -\frac{4M^2\Lambda}{3}\,.$$

This gives

$$C \approx \frac{1}{4M^2}\,, \qquad \frac{\Lambda}{3} \approx -\frac{2}{5M^2}\,.$$

So the effective cosmological constant is negative, what corresponds to the effective **AdS** space,

$$F(r) = 1 - \frac{2M}{r} - \frac{2r^2}{5M^2}$$

inside the shell betwee the photons and **Schwarzschild** spheres

$$2M \le r \le 3M.$$

Geodesic Equations

Geodesic equations for waves can be obtained using the **Lagrangian**

$$L(x^\alpha) = \frac{1}{2} g_{\alpha\beta} u^\alpha u^\beta \ .$$

Two conserved components of 4-momentum are:

$$\frac{\partial L(x^\alpha)}{\partial u^t} = u_t F(r) = E \ , \qquad \frac{\partial L(x^\alpha)}{\partial u^\varphi} = u_\varphi r^2 = J \ ,$$

where E and J denote the waves energy and angular momentum and $u_t = dt/ds, u_\varphi = d\varphi/ds$ are 4-velocities. For the isotropic geodesics,

$$2L = F u_t^2 - \frac{u_r^2}{F} - r^2 u_\varphi^2 = 0 \ ,$$

we find the first integral of the only independent geodesic equation

$$u_r^2 = E^2 - \frac{J^2}{r^2} F(r) = E^2 - U^2 \ ,$$

where

$$U^2 = \frac{J^2}{r^2}\left(1 - \frac{2M}{r} - \frac{2r^2}{5M^2}\right) = U_{Sch}^2 + \frac{2J^2}{5M^2}$$

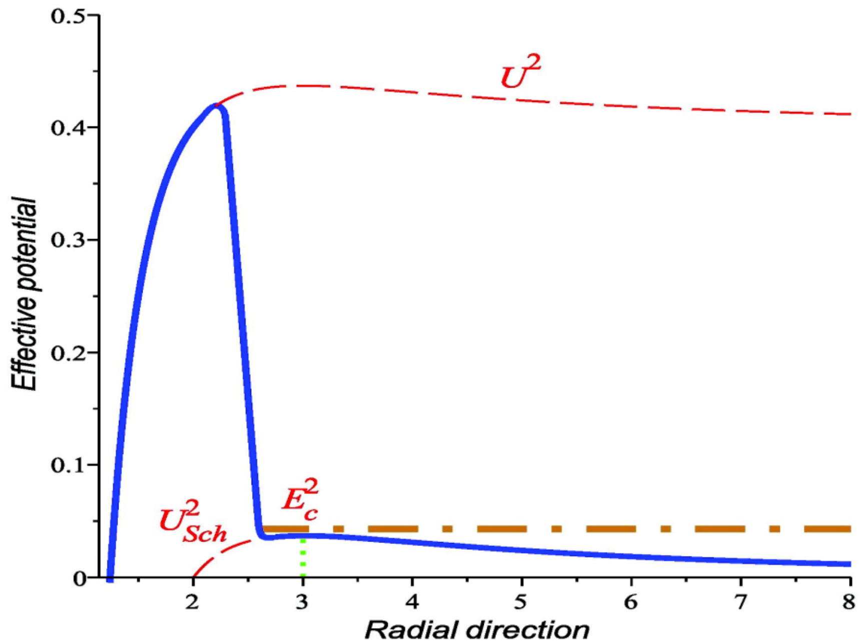
denotes the effective potential.

Effective Potential

The effective potential $\boldsymbol{U^2}$ differs from **Schwarzschild**'s one by $\boldsymbol{2J^2/5M^2}$ and its zero, $\boldsymbol{r \approx 1.2M}$, is shifted inside the **Schwarzschild** sphere.

A non-radial isotropic geodesics with the energy above the critical value, $\boldsymbol{E^2 = J^2/27M^2}$, could cross the photons sphere and fall towards the center of **BH** following the spiral trajectory. However, inside the shell $\boldsymbol{2M \leq r \leq 3M}$ waves cross the effective **AdS** space, where the actual impact factor, $\boldsymbol{E^2/J^2 - 2/5M^2}$, changes the sign. In this region the effective potential forms the second barrier and waves could loop in the minimum between two pics.

Then the waves with decreasing amplitude will lose energy and plunge into singularity. While the photons with increasing amplitude will be reflected from the effective **AdS** space and a distant observer will detect burst-like short signals from the **BH** edge, mainly in gamma and radio frequencies which could escape surrounding the **BH** dust clouds.



Strong GW Lensing

In the case of **GW**s there is chance to observe also relativistic lensing and we need to explore the radial geodesic equation:

$$\dot{r}^2 = F^2\left(\frac{27M^2F}{r^2} - 1\right),$$

overdots mean derivatives by t. Inside $2M \le r \le 3M$ we can assume

$$F(r) \approx \frac{2r^2}{5M^2}.$$

Then for the inspiraling **GW**s we find:

$$r \approx 3M - 11t, \qquad \omega = \dot{\varphi} = \frac{11}{10M} + \frac{9t}{5M^2}.$$

When the effective impact parameter changes the sign, spiraling **GW**s will start deflecting and a distant observer will receive periodic chirp like signals with the exponentially amplified amplitudes and increasing frequency. The expression for the timing of the resulted strain,

$$h \sim A\, e^{t/2M} \sin\omega t + B e^{-t/2M} \cos\omega t\,,$$

can be described by a linear superposition of several quasi-normal modes of perturbed **BH** horizon and these **BH** ringing will imitate the **LIGO** signals.

Conclusions:

- Close to a BH horizon wave equations in the Schwarzschild coordinates have the real-valued exponentially time-dependent solutions.
- For a distant observer these exponential enhancement (decay) of amplitudes will be visible as if the gravitationally lensed waves are receiving (giving) the energy from (to) the BH.
- To model isotropic geodesics in the strong gravitational field between the Schwarzschild and photon spheres we introduced the effective negative cosmological constant with the value that guaranties validity of the eikonal approximation in this region.
- We found that part of the incident waves crossing the effective AdS space below the photon sphere can be amplified and reflected and this mechanism can explain some GWs, GRBs and FRBs.

References:

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გმადლობთ
ყურადღებისათვის